

BUDEM: an urban growth simulation model using CA for Beijing metropolitan area

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ABSTRACT

It is in great need of identifying the future urban form of Beijing, which faces challenges of rapid growth in urban development projects implemented in Beijing. We develop Beijing Urban Developing Model (BUDEM in short) to support urban planning and corresponding policies evaluation. BUDEM is the spatio-temporal dynamic model for simulating urban growth in Beijing metropolitan area, using cellular automata (CA) and Multi-agent system (MAS) approaches. In this phase, the computer simulation using CA in Beijing metropolitan area is conducted, which attempts to provide a premise of urban activities including different kinds of urban development projects for industrial plants, shopping facilities, houses. In the paper, concept model of BUDEM is introduced, which is established basing on prevalent urban growth theories. The method integrating logistic regression and MonoLoop is used to retrieve weights in the transition rule by MCE. After model sensibility analysis, we apply BUDEM into three aspects of urban planning practices: (1) Identifying urban growth mechanism in various historical phases since 1986; (2) Identifying urban growth policies needed to implement desired urban form (BEIJING2020), namely planned urban form; (3) Simulating urban growth scenarios of 2049 (BEIJING2049) basing on the urban form and parameter set of BEIJING2020.

Keywords: cellular automata, policy simulation, urban growth simulation, logistic regression, BUDEM, MonoLoop, desired urban form

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1. INTRODUCTION

Under the background of the boom of domestic macro-economy and Olympic economy, it is in great need of identifying the future urban form, including post-Olympic games (2009), the end year of urban master planning (2020), and the hundredth anniversary of the founding of P. R. China's capital (2049). Furthermore, it is essential to forecast the long-term urban form to support the next round of urban planning. At the same time, in some major metropolitan areas, such as London metropolitan area, San Francisco bay, California area, comprehensive urban models have been developed to simulate urban systems. Yet, this is still vacant in Beijing, even in most China's main cities. Hence, we develop Beijing Urban Developing Model (BUDEM in short) to support urban planning and corresponding policies evaluation. BUDEM is a spatio-temporal dynamic model for simulating urban growth in Beijing metropolitan area, using cellular automata (CA) and Multi-agent system (MAS) approaches. BUDEM is developed for the municipal government and planning committee of Beijing metropolitan area with an area of 16410 km², and its spatial resolution is 500m. In this phase, the computer simulation using CA in Beijing metropolitan area is conducted, which is attempting to provide a premise of urban activities including different kinds of urban development projects for industrial plants, shopping facilities, houses, etc.

Urban models, which are initiated in the early years of twentieth century, have experienced structural models, static models and dynamic models phases. Traditional urban models, basing on differential equations or quasi dynamic, usually simulate urban system in macro level, and lack reflect on the dynamic, self-organizing, emerging characters of urban system in some degree. With the development of GIS and complex science, the urban model, based on artificial life or discrete dynamics, is the hot point in researches. In recent years, it is prevailing to simulate urban growth by means of CA, which is based on self-organizing theory and different from the system dynamic model. It is composed by a series of

basic rules, instead of strictly defined physics equations or functions. Besides, discrete is the key character for the time, space and status in CA. CA is adapted to simulate the emergence, self-organizing and chaos phenomenon in the urban system.

Nowadays, CA is a practical tool to simulate urban growth which is also a main field for its application. Tober(1970) initially simulates urban expansion of five lakes region of America. Couclelis(1985, 1988, 1989, 1997) claims that it is possible for CA with simple rules to generate complex urban forms in a virtual city, and it shows the great potential of CA to be able to be applied in urban growth field. White and Engelen(1993) attempts to apply CA in urban planning, and White and Engelen(1997) simulated the land use pattern of Cincinnati of America. Clark and Gaydos(1998) establish SLEUTH model, and apply it to simulate long-term urban growth of San Francisco bay area and Washington-Baltimore area of America, which is the earlier practice of CA to simulate real cities. Besides, Batty(1991, 1994, 1996, 1997, 1998, and 1999) conduct abundant researches on fractal and CA to study urban formation and expansion phenomenon. Xie(1994) simulates the land use change of Buffalo of America. Wu(1998a, 1998b, and 2002) employs MCE to found the status transition rule of CA, and applies it in the urban expansion simulation of Guangzhou, China.

Regarding domestic relevant applications of CA, Li and Yeh explore various intelligent methods to retrieve transition rules of CA, bring forward constrain CA model to simulate sustainable urban form of Pearl River Delta, and develop uncertainty analysis of CA (Li and YEH, 1998, 2000, 2002, and 2004; YEH and Li, 2001, 2002, and 2006). In brief, other academicians separately employ CA to simulate urban growth of Haikou city (Wu and Zhao, 2002)), Wuhan main city (Luo, 2005), Fuzhou city (Li, 2005), Xi'an city (Guo *et al*, 2004), Northern China (He *et al*, 2006), Partial Beijing metropolitan area (Shen *et al*, 2007).

In summary, there are many applications of CA to simulating urban growth, and in China, CA is also employed to predicting urban growth in some cities. However, there is no report about simulating urban growth of Beijing metropolitan area using CA. BUDEM, as an urban form simulating platform, is tightly developed for Beijing urban planning practice, and is a tempting practice of CA for urban planning application to a mega-city. In BUDEM, logistic regression and MonoLoop are integrated to retrieve transition rule of CA, which is also an innovative method to realize desired urban form. Regarding spatial factors considered in BUDEM, comprehensive environmental constraining, urban planning conditions are adopted to reflect the China's urban developing characters. Moreover, BUDEM is employed to simulate a planned urban form (BEIJING2020) and predict a long-term unknown urban form (BEIJING2049). For this, historical analysis of different phases and model estimation for BEIJING2020 are conducted, and finally BEIJING2049 is predicted based on the urban form and parameter set of BEIJING2020.

2. BUDEM - URBAN SIMULATION MODEL BASING ON CA

2.1 Spatial factors selection

The urban growth researches in macro level, which do not consider urban spatial distribution, regard the urban system as one whole entity, and the drives for urban growth consist of population changing, SciTech varying, economy increasing, political structure, etc (Stern, 1992). However, the paper prefers the self-organizing process research inner urban system. It is clear that in a classical urban land use model urban development is influenced by location and geographic conditions. In his single center urban location theory, Alonso (1964) points out that the distance to urban center is a principal factor of the structure of urban land use. The optimum land type will change according to the distance to urban center, because of the change of the accessibility and transportation cost. In addition, Doxiadis, who founded human settlement science, concluded that the distance to present urban center, to main road, and to natural landscape are the main forces of human settlement (WU, 2001). In addition, Hedonic model provides clearer framework, which considers that commodity price is determined by the total utility of different properties, and price differs with different numbers and composition of commodity's properties (Lancaster, 1966). For example, Butler (1982) holds that residential price is affected by three types of factors, location, architecture structure and neighborhood, and its price reflects the total preferences by the consumer. Urban development is quite familiar with residential price, and its probability is the reflection of relevant parameters of the lot or block by developer. Hence, we choose spatial variables in CA as shown in table 1 (Column "Dataset" stands for the corresponding dataset from which the spatial variable is retrieved, and detailed description on dataset is shown in section 2.4 "Dataset"), referring the theory of Hedonic and considering the acquiring possibility of corresponding data.

Table 1. Spatial variables in CA

Type	Name	Value	Description	Dataset
SELF-STATUS	<i>isrural</i>	0 , 1	Whether the cell is rural built-up land in last iteration	LANDUSE, LANDi*
	<i>isagri</i>	0 , 1	Whether the cell is agricultural land in last iteration	LANDUSE
LOCATION	<i>d_tam</i>	≥ 0	Minimum distance to Tian'anmen square	LOCATION
	<i>d_vcity</i>	≥ 0	Minimum distance to VIP new city	
	<i>d_city</i>	≥ 0	Minimum distance to new city	
	<i>d_vtown</i>	≥ 0	Minimum distance to VIP town	
	<i>d_town</i>	≥ 0	Minimum distance to town	
	<i>d_river</i>	≥ 0	Minimum distance to river	
	<i>d_road</i>	≥ 0	Minimum distance to road	
GOVERNMENT	<i>d_bdtown</i>	≥ 0	Minimum distance to town boundaries	BOUNDARY
	<i>Planning</i>	0, 1	Whether planned as urban built-up	PLANNING
	<i>con_f</i>	0, 1	Whether in forbidden zone	CONSTRAIN
	<i>Landresource</i>	1-8 (integer)	Land classification for agricultural suitability	LANDRESOURCE
NEIGHBOR	<i>Neighbor</i>	0-0.125	Developing intensity in neighborhood	LANDi

* Dataset LANDi is the generated land use layer when CA iterating

2.2 Concept model

The premises of BUDEM establishing are as follows. Firstly, urban is a complex self-adaptive system, and bottom-to-up method is available to simulate urban growth. Secondly, urban growth forces can be classified into promoting type and restraining type. Meanwhile, they can be divided into market driving and government inducing types. Thirdly, the historical developing rule is also fit for the future one with the same developing trend. Finally, various urban growth scenarios can be generated with some modification basing on baseline predicting scenario.

With the premises above, Concept model of BUDEM using CA is established (Shown in formula 1). CA lattices are Beijing metropolitan area, with an area of 16410 km², which is adjustable according to the simulating purpose. Cell size of CA is 500m*500m with the shape of square, and totally 65628 cells are contained in the whole lattices. Cell states of CA is 1 or 0, while 1 stands for urban built-up land, and 0 stands for non urban built-up land. Multi-criteria evaluation (MCE in short) is applied as transition rule of CA, and 3*3 Moore is used as CA neighborhood with 8 adjacent cells. Regarding discrete time of CA, we use the total number of urban built-up cells to link the relationship between it and the real time.

$$\begin{aligned}
 V_{i,j}^{t+1} &= f\{V_{i,j}^t, Global, Local\} \\
 &= \{V_{i,j}^t, LOCATION, GOVERNMENT, NEIGHBOR\} \\
 &= f\left\{ \begin{array}{l} V_{i,j}^t, isrural_{i,j}, isagri_{i,j} \\ d_tam_{i,j}, d_vcity_{i,j}, d_city_{i,j}, d_vtown_{i,j}, d_town_{i,j}, \\ d_river_{i,j}, r_road_{i,j}, d_bdtown_{i,j}, \\ planning_{i,j}, con_f_{i,j}, landresource_{i,j}, \\ neighbor^t_{i,j} \end{array} \right\} \quad (1)
 \end{aligned}$$

$V_{i,j}^t$ is the cell status at ij of time t

$V_{i,j}^{t+1}$ is of the cell status at ij of time $t+1$

f is the transition rule

On the whole, the cell status in each iteration is influenced by self status variables of last iteration, global variables and local variable as listed in table 1. Self status variables include *isrural* and *isagri*, standing for whether cell *i, j*'s status in time *t* is rural built-up land / agricultural land. It needs to be mentioned that the developing control for rural built-up land and agricultural land is sensitive in China. Therefore, we introduce the two self status variables into BUDEM. With respect to global variables, location type and government type variables are included, which keep static among all iterations. Variable *neighbor* is the only local variable in CA, and it keeps changing when iterating. In BUDEM, the transition from urban non built-up to urban built-up is simulated, while the reversed process is not considered, and the urban redevelopment is also not reflected.

Besides CA model, we develop Macro-restraining sub-model (MSM in short) as the other part of BUDEM. MSM, using urban developing serial data as described in "Data" section, is capable of predicting the total amount of urban built-up of each future year. The total amount is the core parameter for controlling the number of CA iterations, namely the time point of model terminating. In addition, it can calculate the corresponding real time (Year) for each step of model simulating (discrete time).

Urban Economists adopt regression method, such as logistic regression, multi-logit model, etc., to calculate the urban developing probability, and it is proved to be one reasonable method to identify the relationship between land use change and locational characters. However, the self-organizing process of land development is not considered. Spontaneous growth and self-organizing growth are integrated in BUDEM using CA and this is its leading merit to simulating urban growth.

2.3 Status transition rule

Transition rule, as the core of CA, has been being the hot point in the field of CA research. Various methods to retrieve transition rule are developed, such as MCE, grey theory, principal component analysis (PCA), artificial neural network (ANN), genetic algorithm, rough sets, case-based reasoning, etc. MCE is implemented to establish the CA status transition rule in this paper.

On one hand, Landis(1994, 1995, 1998a, 1999b) develops CUF and CUF-2 to predict urban form, which is the typical application of MCE to urban growth modeling field. CUF and CUF-2 use DLU (Developing land use unit) as the basic modeling unit, instead of cell of CA. Regarding simulating urban growth using CA with MCE, Wu(2002) brings forward the transition rules as shown in formula 2. And in Yeh and Li(2001)'s constrain CA model, the transition rule is similar with it.

$$P_c^t = P_g * con(s_{ij}^t = suitable) * \Omega_{i,j}^t \quad (2)$$

In the above formula, P_g is the urban growth suitability, namely global probability, calibrated by MCE method, Ω is the neighborhood effect, *con* is the environmental restraining effect, and P_c^t stands for joint probability. Wu(1998a, 1998b) adopts AHP method to obtain the weights of spatial variables in MCE, and Wu(2002) avails logistic regression to retrieve the weights basing historical developing data. Wu finds out one essential and convenient method, namely MCE, as applicable CA transition rule, yet the means of acquiring weights for spatial variables in MCE is concerned by us. AHP method is lack of repeating and also over subjective, and it is impossible to identify the historical urban developing trend. With respect to the latter method for obtaining MCE weights, neighborhood and environmental effects are separately multiplied to P_g , instead of being included in the logistic regression procedure. As a result, the logistic regression does not include all the relevant factors. Therefore the weights via regression can't entirely explain the urban growth developing trend in some history phase.

On the other hand, Clark and Gaydos(1998) put forward rigorous calibration method. In the method, firstly, generate simulating result with different parameter sets (Nested loops). Secondly, match each simulated result with observed form, and calculate the matching indexes (the r-squared fit between the actual and predicted number of Urban pixels, Edges in the images, Separate clusters and A modified Lee-Sallee shape index). Finally, the parameter set with best matching index is then employed to predict urban form in the future. Five parameters are considered in Clark's model, and respectively 6, 6, 6, 5, 7 values for each parameter are tested. Consequently, 7560 parameter sets are generated and used to calibrate, and it takes 252 hours for the rigorous calibration process. If the number of parameters increased, the calibrating time cost will go up exponentially. With regard to BUDEM, there are 14 parameters in it. Supposing that every parameter has 6 values for testing and the same time cost for every iterating, the total calibrating time cost will probably be 298189 years, not to speak of the limitation of only 6 choices for each parameter. In spite of the fact that

Clark and Gaydos' method can identify the most possible parameter set for model simulating, the time cost for such an intensive computing will not be acceptable even calibrated by the most advanced workstation.

We integrate the methods of Wu(2002) and Clark and Gaydos(1998), with each advantage, to retrieve the weights in MCE formatted transition rule of BUDEM (Shown in formula 3). In BUDEM, all the spatial variables, except *neighbor*, are included in the logistic regression equation, and the corresponding coefficients, namely weights w^{1-13} in MCE, can be obtained. In the regression, dependent variable is 1 or 0, since the land use changing status is either "Developed" or "Undeveloped". Independent variables in regression are 13 spatial variables (all except *neighbor*). The dependent variable is acquired by algebra operation on LANDUSE datum of start year and end year. Basing sample tool in ESRI ArcGIS, the dependant variable and independent variables for logistic regression are sampled into a table, which is then analyzed in SPSS environment to obtain w^{1-13} in form of coefficients.

$$\begin{aligned}
 s_{ij}^t &= \beta_0 + \beta_1 * isrural_{ij} + \beta_2 * isagri_{ij} \\
 &+ \beta_3 * d_tam_{ij} + \beta_4 * d_vcity_{ij} + \beta_5 * d_city_{ij} + \beta_6 * d_vtown_{ij} + \beta_7 * d_town_{ij} \\
 &+ \beta_8 * d_river_{ij} + \beta_9 * r_road_{ij} + \beta_{10} * d_bdtown_{ij} \\
 &+ \beta_{11} * planning_{ij} + \beta_{12} * con_f_{ij} + \beta_{13} * landresource_{ij} \\
 &+ \beta_{14} * neighbor_{ij}^t \\
 p_g^t &= \frac{1}{1 + e^{-s_{ij}^t}} \\
 p_{ij}^t &= \exp \left[\alpha \left(\frac{p_g^t}{p_{g\max}^t} - 1 \right) * RI_{ij}^t \right] \\
 RI_{ij}^t &= 1 + (\gamma_{ij}^t - 0.5) / k \\
 \text{if } p_{ij}^t &> p_{threshold} \text{ then } V_{ij}^{t+1} = 1
 \end{aligned} \tag{3}$$

After that, the weigh for variable *neighbor* (wN^*) can be calibrated out, by using sole parameter looping method (MonoLoop), instead of looping all parameters' weights like Clark and Gaydos. Various wN values are calibrated to find wN^* with the best matching index while keeping obtained w^{1-13} constant. Then, wN^* retrieved accomplishing with w^{1-13} are inputted in the transition rule to simulate urban growth form. *Goodness-of-fit* (accuracy of point to point comparing, G in short) is selected to assess the matching degree between simulated and observed urban forms, and its maximum value in theory is 100%. For one thing, our method combining logistic regression and MonoLoop can reduce model calibrating time greatly. For another thing, the method is able to identify historical urban growth trend.

In formula 3, s_{ij}^t stands for developing suitability, β , coefficients in logistic regression, p_g^t , initial transition probability, $p_{g\max}^t$, the max value of p_g^t in iteration t , p_{ij}^t , final transition probability, $p_{threshold}$, urban growth threshold, RI , random item, γ , random value varying from 0 to 1, k stands for random index, used to regulate RI , and α stands for dispersion parameter ranging from 1 to 10, indicating the rigid level for developing. The bigger α is, the stricter the developing control is, namely, the smaller developing probability with the same suitability. Hence, the parameter α has a great impact on the simulated urban form.

Moreover, random item (RI) is included in transition rule to denote the developing out of the explanation of selected spatial variables, such as leapfrog type developing. With introducing RI into the transition rule, the simulated result will be more practicable. Constant threshold ($p_{threshold}$) is applied, instead of static or random one, which can guarantee the same developing standard for urban develop in different phases. Meanwhile, variables *isagri* and *isrural* stand for the transition from agriculture land and rural built-up, which is concerned by Beijing municipal government.

2.4 Dataset

We classify spatial data in BUDEM into seven types, LANDUSE, PLANNING, CONSTRAIN, LANDRESOURCE, LOCATION, BOUNDARY, as well as UrbanInfoSeries. All spatial data are converted into ESRI single band GRID format, with the same coordinate and projection system.

(1) LANDUSE: as the most complete data ever since the foundation of China, the time points of landuse cover the years of 1947, 1964, 1976, 1981, 1986, 1991, 1996, 2001 and 2006, in which, data of year 1947 is digitalized from relief map,

data of year 1964 is interpreted from DISP aerial image, data of year 1976 and year 1981 are interpreted from MSS images, and others are interpreted from TM images. LANDUSE is classified into six land use types, urban built-up area, rural built-up area, agriculture area, forest area and vacant area. Variable *landuse* is retrieved from LANDUSE data, and it is valued 1 for urban built-up area, and 0 for other areas.

(2) PLANNING: five times urban master plans for Beijing metropolitan area were conducted, and currently it covers the years of 1958, 1973, 1982, 1993 and 2004, which is classified into urban built-up area and non urban built-up area (Beijing urban planning committee et al, 2006). Variable *planning* is retrieved from PLANNING.

(3) CONSTRAIN: it reflects the urban developing constraining degree, and is generated by 110 spatial layers of natural resources protection and hazard prevention according to current laws, legislations, and standards of China. CONSTRAIN is zoned into three types: forbidden built-up area, constraining built-up area and suitable built-up area. Variable *con_f* stands for forbidden built-up area.

(4) LANDRESOURCE: it stands for the suitability of agricultural land use, which is classed into eight types ranging from 1 to 8, showing the suitability in turn (Beijing planning commission, 1988). Variable *landresource* is obtained from LANDRESOURCE with the same value.

(5) LOCATION: it stands for the location condition, including the minimum distance to urban or town centers of different administrative levels, to road (*d_road*) and to river (*d_river*). Location type spatial variables are retrieved from LOCATION using Distance/Straight Line command in ESRI ArcGIS.

(6) BOUNDARY: it consists of administrative, ring road, eco-zoning, as well as watershed boundaries, and different transition rules in various areas are available within the data (not conducted in this paper). Variable *d_bdtown* is retrieved from the data.

(7) UrbanInfoSeries: population, resources, environment, economy and society data are included in UrbanInfoSeries, which favors the Macro-restraining sub-model (Beijing statistic bureau, 1999).

3. PARAMETER ESTIMATION IN BEIJING USING BUDEM

Urban growth rules in different historical phases can be acquired by logistic regression, including 1947-1964, 1964-1976, 1976-1981, 1981-1986, 1986-1991, 1991-1996, 1996-2001, and 2001-2006. In logistic regression, variables *d_tam*, *d_vcity*, *d_city*, *d_vtown*, *d_town*, *d_bdtown*, *landresource*, and *con_f* do not change in different phases; yet, variables *planning*, *d_road*, *isrural*, *isagri* and dependent variable vary from historical phases. In addition, variable *neighbor* is not considered in logistic regression. Through the process, it is available to compare urban growth mechanisms in various phases. Regression result of 2001-2006 is listed in table 2, and the accuracy of regression is acceptable 88.5%. As is shown in the table, the coefficient of *d_river* is the biggest one, and accordingly the developing probability will decrease by 0.024%, if the minimum distance to river increases 1 meter. Regarding *d_road*, it is positively correlated to the developing probability, and it is clear that the urban growth along roads in this phase is not evident. Besides, the developing probability of the cell located in the forbidden built-up zone is lower by 65.9% than that of other areas, and the cell in planned built-up area has a developing probability of 36.7% higher than that of unplanned area.

Table 2. Variables in the equation of logistic regression of 2001-2006

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1	d_tam	-.000042	.000	68.054	1	.000	1.000
	d_vcity	-.000018	.000	10.018	1	.002	1.000
	d_city	-.000032	.000	26.877	1	.000	1.000
	d_vtown	-.000019	.000	3.829	1	.050	1.000
	d_town	-.000036	.000	4.590	1	.032	1.000
	d_river	-.000224	.000	41.815	1	.000	1.000
	d_bdtown	-.000057	.000	.434	1	.510	1.000
	con_f	-1.076304	.143	56.624	1	.000	.341
	landresource	-.023686	.040	.347	1	.556	.977
	isrural	3.284309	.321	104.865	1	.000	26.691
	isagri	4.376267	.157	781.214	1	.000	79.541
	d_road2001	.000061	.000	.310	1	.578	1.000
	planning2004	.312422	.139	5.043	1	.025	1.367

The coefficients of logistic regression in different historical phases are shown in table 3 (LANDUSE data before 1986 is under processing. Therefore, corresponding logistic regressions are not conducted to data.). Comparing the dominant

factors in different phases, it is clear that these urban growth mechanisms differ from each other to a great degree; in addition, market and government role also vary in different phases. In 2001-2006, riverside developing is notable, and then is the central city, while road along type is not notable. In 1996-2001, road along developing is notable, and then is new city, while areas around town grow at a low speed. In 1991-1996 and 1986-1991, road along developing is notable.

Comparing the variation of factors in different phases, according to the table 3, with regard to $w^{planning}$, it keeps positive, and reaches its maximum value in 1986-1991. In others phases, it does not change greatly. We can see that in the first several years of social market economy, urban planning played a leading role in urban growth. However, with the introduction of market mechanism into China, its role is being partially replaced by market factors. Regarding w^{con-f} , it remains negative, and its absolute value continually increases, which means that its role in urban growth control is increasing, and the strengthen for ecological space protection and hazard prevention is also going up. Additionally, $w^{landresource}$ keeps positive with its absolute value declining, and it shows that the protection intensity for more cultivating suitable area is descending. Moreover, w^{d_bdtown} is positive before 1996, and is negative after that. It can be concluded that the restricting action by administrative boundary is gradually fading in Beijing.

Table 3. Coefficients of logistic regression in different historical phases

variable	B(2001-2006)	B(1996-2001)	B(1991-1996)	B(1986-1991)
w^{d_tam}	-0.000042	-0.000049	-0.000054	-0.000012
w^{d_vcity}	-0.000018	-0.000032	0.000003	-0.000047
w^{d_city}	-0.000032	-0.000094	-0.000034	-0.000028
w^{d_vtown}	-0.000019	-0.000029	-0.000018	-0.000014
w^{d_town}	-0.000036	0.000129	0.000023	-0.000021
w^{d_river}	-0.000224	-0.000078	-0.000066	-0.000021
w^{d_road}	0.000061	-0.000734	-0.000365	-0.001232
w^{d_bdtown}	-0.000057	-0.000463	0.000001	0.000182
$w^{planning}$	0.312422	0.459742	0.416635	1.302933
w^{con-f}	-1.076304	-0.613198	-0.449983	-1.274498
$w^{landresource}$	-0.023686	-0.140539	-0.131834	-0.350835
$w^{isrural}$	3.284309	-3.774535	-3.949259	-5.534083
w^{isagri}	4.376267	3.279759	1.802018	-0.156322

4. A SOLUTION FOR PLANNED FORM USING BUDEM

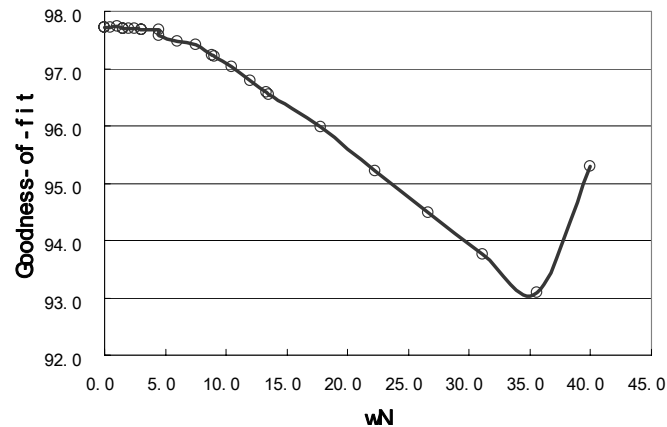
In 2004, State Council of China approved Beijing master planning (2004-2020). How to realize the planned urban form (represented by Data PLANNING2004), and what the possible urban form will be in years from now to 2020, concern the Beijing municipal planning committee and even Beijing municipal government. Regarding the issue of simulating desired urban form, it is impossible to retrieve best parameter set via methods of differential equation or optimum theory, since CA model is one completely non-linear method. Moreover, in above paragraphs, it is discussed for nest-loop method failing to avail it. Here we employ the integrating logistic regression and MonoLoop method, as described in “BUDEM - urban simulation model basing on CA” section, to retrieve corresponding parameter set to establish the transition rule for desired urban form.

To begin with, logistic regression is conducted. The dependent variable is calibrated via algebra operation on PLANNING2004 and LANDUSE2006, and then w^{I-13} can be obtained by means of logistic regression (regression result as shown in table 4, accuracy of regression is as high as 96.8%). According to the table below, the significance of each independent variable is at the acceptable level.

Table 4. Variables in the equation of logistic regression in 2006-2020

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1	isrural	6.88621	.311	489.806	1	.000	978.683
	isagri	6.97187	.212	1081.601	1	.000	1066.213
	d_tam	-.00010	.000	351.190	1	.000	1.000
	d_vcity	-.00003	.000	22.971	1	.000	1.000
	d_city	-.00010	.000	242.798	1	.000	1.000
	d_vtown	-.00028	.000	512.563	1	.000	1.000
	d_town	-.00011	.000	46.814	1	.000	1.000
	d_river	-.00052	.000	162.656	1	.000	.999
	d_road	.00096	.000	110.505	1	.000	1.001
	d_bdtown	-.00027	.000	10.079	1	.001	1.000
	planning	8.77071	.270	1055.034	1	.000	6442.743
	con_f	-.20097	.138	2.111	1	.146	.818
	landresource	-.09355	.039	5.850	1	.016	.911

Then, the logistic regression is followed by MonoLoop procedure. We attempt 27 values for wN , and it costs 21.5h for 2997 iterations totally. The relationship between G and wN is shown in figure 1. In the figure, G keeps steadily being around 97.6% when wN ranges from 0 to 5, and G decreases sharply to 93.0% when wN is within 5 and 35, and G rises when wN is bigger than 35. Moreover, if wN is bigger than 35, the number of developed cells in the first iteration is too large (that is to say that the neighborhood action is too big), even exceeding the total number of destined built-up cells, to fulfill the simulation. Therefore, we choose wN^* as 4.59808 to simulate BEIJING2020, and correspondingly, G will be around 97.6%, which is near the ideal maximum value 98.9% (Regarding the ideal maximum value, in the planning scheme of PLANNING2004, the number of urban built-up cells is 9376, while in the observed urban form of LANDUSE2006, the number of urban built-up cells is 5297. Through overlay analysis on PLANNING2004 and LANDUSE2006, 712 cells are the observed urban built-up out of planned form. Hence, the ideally maximum of G should be equal to $(65628-712)/65628=98.92\%$).

Fig. 1. dG/dt of MonoLoop procedure in BEIJING2020

Weights w^{1-13} obtained by logistic regression and wN^* calibrated by MonoLoop are simultaneously inputted into the established transition rule, and then urban form of BEIJING2020 is simulated with 208 iterations and 6297s' time cost. Simulated urban form is shown in figure 2, with 10104 developed cells. It is evident that the simulated urban form and the planned one are greatly consistent. The simulated urban form of 2020, together with that of various phases from now to 2020, can be consulted by urban planning department as pre-awareness or references to regulate corresponding policies.

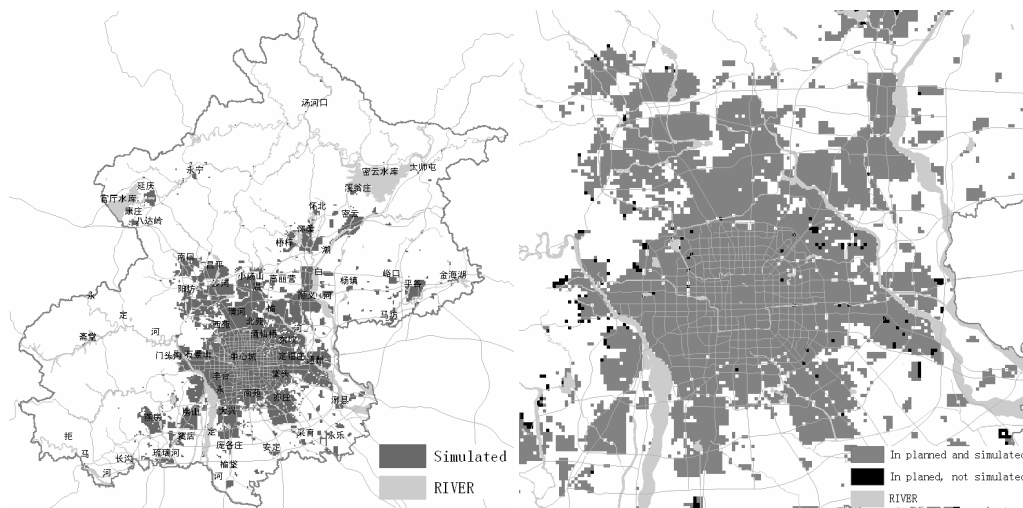


Fig. 2. Simulated urban form of BEIJING2020 (Left) and its comparative image with planned form in Beijing central metropolitan area (Right)

G values during the simulation process of BEIJING2020 are plotted in figure 3, and it shows that the maximum G appears at the iteration of 117 ($G=97.75\%$), instead of at the last iteration 208 ($G=97.57\%$) as presumed. With MRM, iteration 117 corresponds to year 2019. It illustrates that the real realization year of this round of urban planning is 2019, in contrast with 2020. Therefore, it is necessary and timely for Beijing municipal planning committee to carry out the new round of urban planning in 2019, and we should also prepare for the new urban planning in far advance, such as 2015.

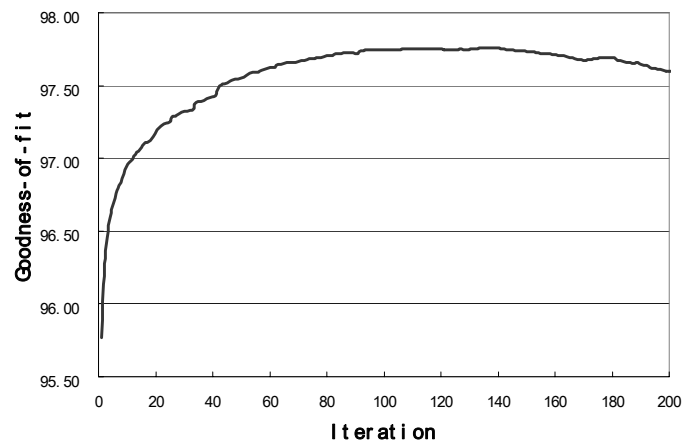


Fig. 3. dG/dt of BEIJING2020 simulation

Accordingly, the urban growth policies to achieve planned urban form can be identified, via comparing the logistic regression coefficients of BEIJING2020 with those of some historical phases. Through contrasting the present policies and achieved policies, we can recognize whether they are consistent, if not, corresponding regulating is available to be brought out by BUDEM. For instance, comparing with the coefficients of 2001-2006, to realize the planned urban form, we should strengthen the intensity of urban planning implementing, emphasize urban growth along the main road, and increase the constructing actions at VIP towns. Moreover, the protection degree for forbidden built-up area does not need to be enhanced. In particular, BUDEM is capable of generating other urban forms in 2020 via adjusting the obtained policies for BEIJING2020, to testify various spatial developing policies advanced by the government.

In contrast with generous CA model for urban growth, point to point comparing validation of CA is conducted during MonoLoop process in the above simulation, and the accuracy of point to point comparing validation is 97.6%. The G is one key constraining condition in MonoLoop of BUDEM, which guarantees the highest model accuracy on point to point comparing. Hence, it shows one of the merits of BUDEM with MonoLoop method. There are many kinds of methods for CA model validation, such as fractal indexes, spatial structure indexes. Besides point to point comparing method, we adopt Moran I index (the degree of spatial autocorrelation) to validate the simulated result of BEIJING2020. The Moran I is equal to 0.12 (Z Score = 31.1) for planned urban form, and is equal to 0.16 (Z Score = 43.6) for simulated one. It is clear that not only simulated but also planned urban form is comparatively concentrated (It is one character for the planned urban form conducted by planners instead of natural growth), while simulated urban form is more congregated than planned one.

5. FUTURE URBAN FORM BASING ON BEIJING2020 SCENARIO USING BUDEM

As is known, 2020 is the ending year of this round of urban master planning drafted by Beijing municipal planning committee, while year 2049 is the hundredth anniversary of Beijing as the capital of P. R. China. To prepare for the next round of urban planning of Beijing, it is necessary to predict the urban form from 2020 to 2049, especially 2049. The prediction is divided into two aspects, one is to forecast the total amount of urban built-up area, and the other is to allocate the forecasted total areas of built-up into different spaces.

It will be more accurate to forecast Beijing urban form of 2049 basing on the planned urban form of 2020 than basing on the observed urban form of 2006, conditioning the higher probability for BEIJING2020 to realize. In China, land developing is controlled by government by means of urban planning to a large degree, and urban planning form can explain most of urban developing activities, which are mostly located in area planned as urban built-up. Therefore, it is reasonable to forecast long-term urban growth using planned urban form of some middle year, which is possible to reduce the uncertainty of long-term forecasting. However, in some leading western countries, land property is greatly charged by private owners, and the urban developing is less controlled by government than by planning department of China. Hence, we forecast urban form of 2049 with the premise of the implementation of BEIJING2020 planned form.

5.1 Sensibility analysis

To forecast the urban form scenarios of 2049, we need to adjust the value of parameters in BUDEM, especially the weights of spatial variables. Therefore, it is necessary to estimate the parameters' sensibility to better understand the influence of the parameter's regulating on model output. Generally, there are two kinds of methods for sensibility analysis, one is analyzing the sensibility of parameters to status and objective, and the other is analyzing the sensibility of status to objective. We choose the first one as the sensibility analysis method, and emphasize the influence of status transition rule on simulating status variables and objective.

In BUDEM, the adjustable parameters are w^{I-I4} , α , k and $p_{threshold}$. Definition of the base parameter set (BPS in short) is the premise for sensibility analysis. In the base parameter set, w^{I-I4} are the same with BEIJING2020, $\alpha=3$, $k=20$, $p_{threshold}=0.99$, and the iteration number is set to be 10, that is to say that the model will terminate at the end of iteration 10. The influence on the simulating result by adjusting various single parameter can be calculated by simulating with changed parameter set. We select five indicators to describe the influence, including x (the number of cells with changed status), $AveSij$ (average of s_{ij} in all lattices), $AvePg$ (average of p_g in all lattices), $AvePij$ (average of p_{ij} in all lattices), and G . The sensibility analyzing procedures are listed as below:

- (1) Simulate with BPS, acquire the simulating result of iteration 10, and calculate each indicator's value $W_i(BPS)$, where $i=1-5$, standing for the ID of indicators.
- (2) Replace parameter j in BPS with multiplying by 0.8 while holding other parameters constant in BPS, and get indicator i 's value $W_i(BPS_j*0.8)$, where $j=1-17$, standing for the ID of parameters.
- (3) Calculate the sensibility of parameter j to indicator i : $U_{ij}=\text{abs}[W_i(BPS_j*0.8) / W_i(BPS)-1]$, where "abs" stands for the absolute value. The result of sensibility analysis is shown in table 5.

Table 5. Result of sensibility analysis

Parameter Name	$U(x)$	$U(AveS)$	$U(AvePg)$	$U(AvePij)$	$U(G)$	$U(SUM)$
A	0.018	0.000	0.005	0.321	0.003	0.347
K	0.001	0.000	0.000	0.001	0.000	0.002
$P_{threshold}$	0.171	0.004	0.088	0.031	0.013	0.308
$w_{isrural}$	0.006	0.004	0.013	0.001	0.001	0.024
w_{isagri}	0.088	0.031	0.103	0.030	0.012	0.265
w_{d_tam}	0.040	0.108	0.138	0.095	0.005	0.385
w_{d_vcity}	0.005	0.022	0.029	0.022	0.001	0.078
w_{d_city}	0.033	0.047	0.075	0.052	0.004	0.212
w_{d_vtown}	0.054	0.067	0.163	0.141	0.007	0.433
w_{d_town}	0.018	0.017	0.042	0.030	0.002	0.110
w_{d_river}	0.023	0.031	0.049	0.036	0.003	0.141
w_{d_road}	0.004	0.041	0.019	0.013	0.001	0.078
w_{d_bdtown}	0.005	0.006	0.008	0.006	0.001	0.026
$w_{planning}$	0.146	0.019	0.210	0.166	0.020	0.562
w_{con_f}	0.002	0.002	0.003	0.002	0.000	0.009
$w_{landresource}$	0.005	0.007	0.010	0.008	0.001	0.031
$w_{neighbor}$	0.013	0.011	0.112	0.100	0.002	0.238
$U(SUM)$	0.633	0.418	1.068	1.055	0.076	

From table 5, the weight of *planning* is the most sensitive, and then is the weight of *d_vtown*, while *k* is the least sensitive. Regarding indicators, *AvePg* is the most sensitive, while *G* is the least sensitive. On the one hand, with respect to indicator *x*, *P_{threshold}* is the most sensitive, and then is the weight of *planning*. On the other hand, with respect to indicator *G*, *planning* is the most sensitive, and then is *P_{threshold}*. In summary, sensibility analysis is the base of model calibration and application, and the model calibrating rule can be identified via such work.

In addition, to better understand the input and output relationship, it is essential to track the status changing trend with iteration number. We select dx/dt , $d(AvePij)/dt$ and dG/dt as indicators for status of the model, store the relevant data in database for iteration 1 to 100, plot and analyze it. Pages limited, the details are not prescribed here.

5.2 BEIJING2049

Formula 4 is used to forecast the total amount of urban built-up of Beijing metropolitan area. As shown in the formula, $a=2344\text{km}^2$ (Total urban built-up area planned in year 2020), $b=30\text{km}^2/a$, $c=2020$, *x* stands for the year number to forecast, and *y* stands for the forecasting result of urban built-up (km^2). Regarding *d* as increasing parameter, it is used to adjust the urban built-up increasing velocity.

$$y = a + b * (x - c)^d \quad (4)$$

After forecasting the total urban built-up area, CA is used to simulate its distribution. In the established transition rule, the value of weight stands for the intensity of corresponding urban growth policy (Valued as 0 denoting that the just policy is not considered). Furthermore, spatial variable itself also stands for urban growth policy, for example, the spatial distribution of *con_f* stands for the extent of protected zone, which is also a policy. Thus, it is possible to simulate different urban growth scenarios for Beijing 2049 by BUDEM by means of adjusting the weights or the variables' spatial distribution, such as urban sprawl scenario, grape-cluster scenario, sustainable scenario, center city promoting scenario, town promoting scenario, riverside promoting scenario, roadside promoting scenario, agriculture protecting scenario, rural built-up controlling scenario, etc. Moron I index, agricultural land encroachment, forbidden built-up area encroachment, rural built-up area covered are available to assess different scenarios and identify the implementing effect of scenarios. Pages limited, we only propose the urban growth scenario with the same trend of BEIJING2020 here ($d=1$),

and the parameter set is the same with that of BEIJING2020. The simulated result is shown in figure 4, from which we can see that urban growth in 2049 engrosses partial forbidden area, and urban sprawl phenomenon also appears. Thus, it is necessary to adjust the urban growth policies from 2020 to 2049, and re-simulate to generate a more sustainable urban form, avoiding such phenomenon.

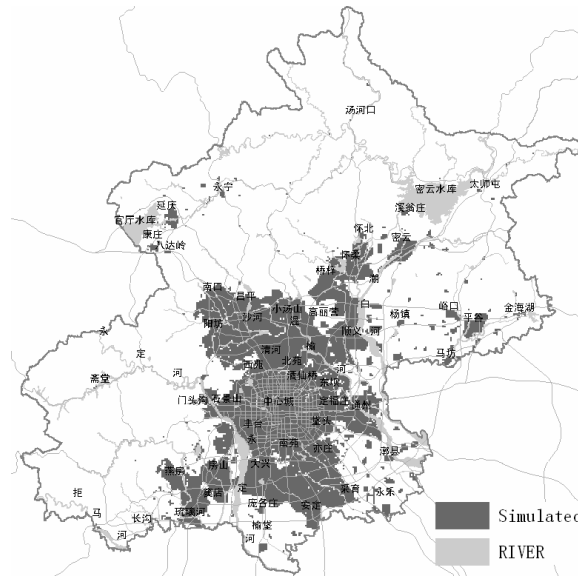


Fig. 4. Simulated urban form of BEIJING2049

6. CONCLUSION

This paper mainly discusses the calibration of CA urban growth simulation model in Beijing metropolitan area in order to find a solution of simulating possible urban form of Beijing in the future. We develop BUDEM as the urban growth simulation platform with sufficiently relevant database, identify the urban growth mechanism of different historical phases, and conduct model sensibility analysis to better understand BUDEM itself, which is also one key work for model calibration and application. In BEIJING2020 section, we propose the method integrating logistic regression and MonoLoop to retrieve the transition rule for desired form. In addition, through comparing the weights obtained with historical ones, the realization policies for urban planning form can be given out. We also attempt to predict urban form in 2049, and BUDEM is capable to simulate different scenarios by various policies sets. CA has been proved as one applicable tool for urban planning issues, and it has played an important role in the practice work, since the urban form confirmation is one core content in urban planning.

As the further work of this phase, we plan to apply BUDEM in amounts of urban planning management and workout practices by means of simulating urban growth. Meanwhile, it is considered to add simulate urban density and competing land use transition functions to apply BUDEM in block scale. For our further research of the second stage of BUDEM, MAS employing discrete choice model will be added into current BUDEM in order to simulate urban agents of different activities based on different scenarios of urban policy.

Note: the simulated form of Beijing metropolitan area is the primary output of BUDEM, which is the authors' self view instead of BICP.

APPENDIX: DATA DESCRIPTIVE STATISTICS TABLE

Variable	N	Minimum	Maximum	Mean	Std. Deviation
<i>d_tam</i>	65628	0	129711	57737.08	26952.19
<i>d_vcity</i>	65628	0	102504	46545.37	23392.81
<i>d_city</i>	65628	0	78824	24801.90	14730.19

Variable	N	Minimum	Maximum	Mean	Std. Deviation
<i>d vtown</i>	65628	0	36530	13255.02	6968.38
<i>d town</i>	65628	0	42005	8286.05	5298.70
<i>d river</i>	65628	0	14230	3212.68	2416.59
<i>d bdtown</i>	65628	0	7762	1239.24	1173.94
<i>d road1986</i>	65628	0	29681	2514.47	3675.36
<i>d road1991</i>	65628	0	29954	2390.63	3577.05
<i>d road1996</i>	65628	0	29820	2341.07	3635.95
<i>d road2001</i>	65628	0	24000	1925.85	2494.29
<i>d road2006</i>	65628	0	29820	2306.49	3613.74
<i>landuse1986</i>	65628	1	6	4.32	1.02
<i>landuse1991</i>	65628	1	6	4.31	1.03
<i>landuse1996</i>	65628	1	6	4.27	1.10
<i>landuse2001</i>	65628	1	6	4.24	1.14
<i>landuse2006</i>	65628	1	6	4.19	1.20
<i>planning1958</i>	65628	0	1	4.28E-02	.20
<i>planning1973</i>	65628	0	1	5.00E-02	.22
<i>planning1982</i>	65628	0	1	2.77E-02	.16
<i>planning1992</i>	65628	0	1	5.16E-02	.22
<i>planning2004</i>	65628	0	1	.14	.35
<i>con f</i>	65628	0	1	.44	.50
<i>landresource</i>	65628	1	8	3.92	1.66
Valid N (listwise)	65628				

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